# How Long Do Stars Live? The Mass-Luminosity Relation

## Course Outcomes Met

1. Communicate scientific issues effectively in oral and written form;
2. Effectively collect, analyze and present data and correctly construct and interpret charts, graphs and tables to draw scientific conclusions;
3. Apply the fundamental concepts and methodologies of physics and/or chemistry to investigate a scientific theme.

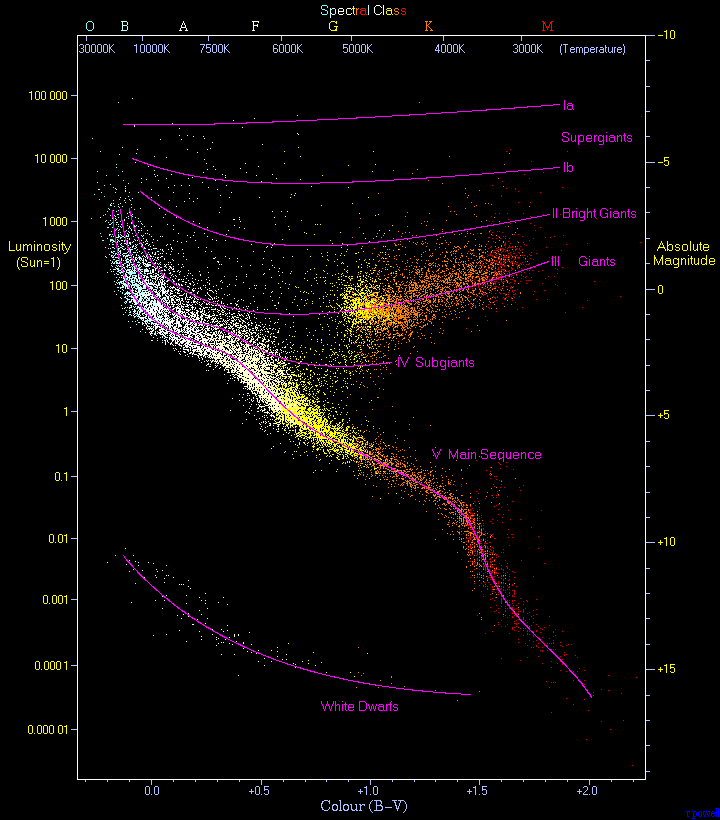
It may or may not surprise you to know that the luminosity of a star is related to its mass. However, from your reading, you should know that for the same star, luminosity varies wildly depending on how old it is due to the size of the star changing radically as it ages. So the mass-luminosity relation is only valid for stars while they are on the main sequence. Nevertheless, it gives us an easy way to estimate the lifetime of stars.

Deriving a theoretically exact mass-luminosity relation requires finding the energy generation equation from the hydrogen fusion rate and building a thermodynamic model of the inside of a star. This is not for the faint hearted. It is well above your pay grade and probably above mine as well. Even the simplified derivation found by Arthur Eddington requires knowledge of advanced calculus and thermodynamics. So we will stick with an empirical model based on the measured properties of stars and call it a day.

## A. Collecting Some Data

Consult the spreadsheet **50 Brightest Stars.** This is the best table I could find that included the radius of each star. We know that a lot of these stars are on the main sequence, but we also know that quite a few are not. Betelgeuse, for instance, is well up in the giant region.

So the first thing we need to do is weed them out. You can do this using a property of stars called the Luminosity Class. This idea is an extension of spectral class. If you look on the HR diagram at a particular spectral class/B-V color index, there are multiple possible luminosities. Suppose you look at color index 1.0 in this diagram:



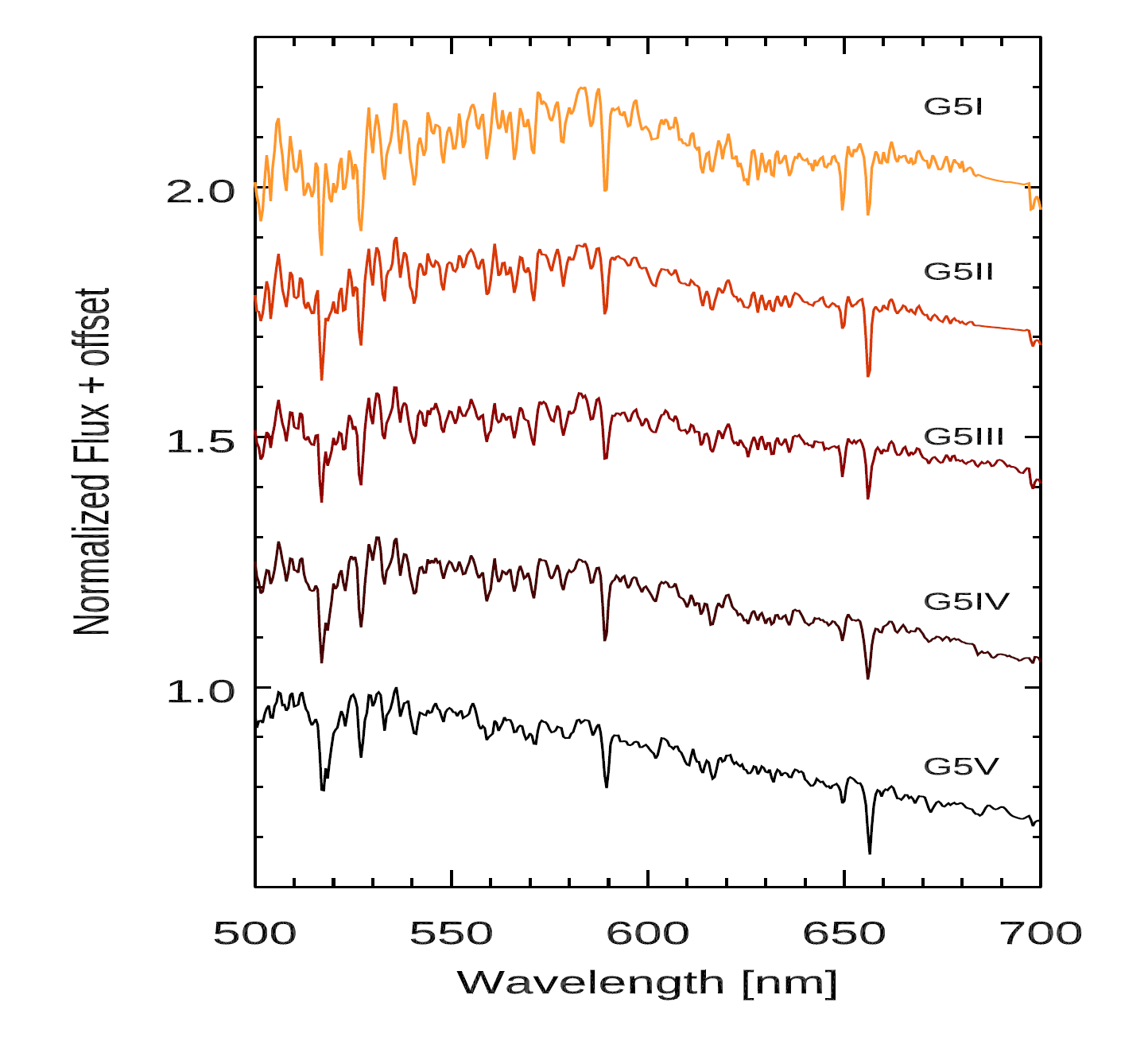
There are seven different possibilities for luminosity class, only one of which is the main sequence.

The spectral classes only account for the star’s spectral characteristics based on its temperature. The luminosity classes account for the star’s spectral characteristics based on its temperature, and the surface gravity effects on a star’s spectrum. Surface gravity is the strength of the star’s gravitational field at the surface. It depends on the distance of the surface from the center, i. e., the radius. The physics is a bit

complicated and the effects are very subtle, so it is a little tricky to determine luminosity classes and we won’t get deeply into it. The short version is as a star expands to become a giant or supergiant star, the surface gravity becomes smaller. This is because the star hasn’t changed mass very much, but now the surface is much further away from the center, so the surface gravity is lower. The lower surface gravity manifests itself in the spectrum of a star in a few ways.

Primarily, larger stars with lower surface gravity have narrower spectral lines due to decreased pressure broadening. That’s one thing we don’t want to get into, but the general idea is that the higher the pressure the more frequently atoms collide and the stronger those collisions are. So those nice, neat atomic energy levels we looked at back in the What Are Stars Made Of activity are made fuzzy as collisions push electrons randomly closer to the nucleus. Fuzzy energy levels means wider absorption lines since electrons can jump orbits from a range of energies to a different range of energies instead of just one specific energy to another specific one. The higher the pressure the bigger the ranges.

But it is definitely not easy to see. Here’s an example. These are all spectral class G5 stars with different luminosity classes, I – V.



1. These may look all identical at first glance but they aren’t. Look, for instance, at the magnesium line near 520 nm. What do you notice about it when you compare the spectra?

2. Take a look at the Hα Balmer line at 653 nm compared to the one to its left at 650 nm. What do you notice about those when you compare the spectra?

These are the kinds of very fine discriminations you have to make to determine luminosity classes, so we won’t delve any further into that.

The Morgan-Kennan luminosity classification divides stars into these types:

* Luminosity Class, V - Main sequence stars
* Luminosity Class, IV – Subgiant stars
* Luminosity Class, III – Giant stars
* Luminosity Class, II – Bright Giant stars
* Luminosity Class, I – Supergiant stars
  + Luminosity Class I is further subdivided into:
    - Ia – Luminous Supergiant stars
    - Ib – Less Luminous Supergiant stars

which you can see on the HR Diagram above. The full class of a star would then be its spectral class and luminosity class. For example, a class K5 giant star would be class K5III.

Stars that are not on the main sequence are not directly comparable. They are reaching the end of their lives so their structures vary as does the means by which they generate energy. So we want to focus on the main sequence, where all the stars have similar structures and they all generate energy by hydrogen fusion.

Luminosity class V is the main sequence, so we want to go through the table and throw out everything else. There are also a few for which the mass or the radius are not well known so we can’t use them either. Some stars are binaries so you may see two values for mass and luminosity. If either or both are luminosity class V, you can use them.

## B. Mass-Luminosity

It will be handy to have a few more data points to get a better trendline. So in addition to the **50 Brightest Stars** spreadsheet, also do the same filtering on the **Nearest Stars** spreadsheet. Unfortunately, I could not find a table that included luminosities for these stars so we will have to calculate it from the absolute magnitude. This is easy enough to do using an equation from Classifying Stars II.

mB - mA = 2.5 log10 (IA / IB).

We can get an equivalent equation for luminosities and absolute magnitudes. These are defined to be values measured at a distance of 10 parsecs from the star. So the intensity ratio should also be the luminosity ratio as the distance factor divides out. So the equivalent equation for absolute magnitudes and luminosities is:

MB – MA = 2.5 log10 (LA/LB)

Solve this for LA/LB, noting that 1/2.5 = 0.4.

If we let star 2 be the Sun, then we get the relative luminosity, just like the 50 Brightest Stars.

By definition, in the scale we are using, the Sun is as bright as itself so LSun = 1. By direct measurement, MSun=4.85. So the equation you need is:

In solar units. I strongly suggest you let Excel do the calculations for you. What you should have in the end is masses and luminosities for class V stars from both spreadsheets.

1. So now you have a set of main sequence stars for which you can plot the luminosity vs. the mass. Note that these are both in solar units so the number is a multiple of the corresponding value for the Sun. If, for instance, mass = 2 that means twice the mass of the Sun. So take your data and make your graph. The range of luminosities is huge, so make your graph big so you aren’t looking at a lot of data points squished together at the bottom.

2. We also know how to do a trend line. So do one for this data. What is your equation?

Discuss your results with your instructor at this point

## C. Star Lifetime

1. Detailed calculations, described above, lead to a mass-luminosity relation of L = Mn where we use solar units and the value of n is usually in the range 3.5 to 4, depending on the model. How does your result compare?

2. Main sequence stars shine by fusing hydrogen into helium. So you might suppose that the lifetime of a star depends on its mass (since some fraction of that is the amount of hydrogen it has available to fuse) and its luminosity (since that depends on how fast the hydrogen is being used up by fusion). So roughly speaking, the lifetime of a star should be how much hydrogen it has divided by how fast it is using that hydrogen. Since we know the mass and luminosity in solar units, we should use the same for the lifetime. So in solar units, the lifetime is about

From your trendline value of n, what is your equation for a star lifetime in solar lifetimes?

3. Use your version of this equation to estimate the lifetimes of the main sequence stars in your table.

4. Of course, all the mass is not hydrogen and even of the hydrogen that is there not all of it will fuse while the star is on the main sequence. A considerable amount of effort has gone into computationally modeling that fraction for the Sun and then from it computing the Sun’s lifetime. The result is about 10 billion = 1010 years. Use this to calculate the age in years of your stars. What range do you see?

5. See if you can find a pattern in the spectral class and the lifetime (measured either way you want). Can you draw a conclusion about the relative lifetimes of blue vs. yellow vs. red stars?

6.How can you tell from a photograph that the Pleiades are a young group of stars? If there were a red star in the Pleiades, would it also be young or would it be old?

7. Can you find a pattern in the mass and the lifetime? Is it surprising?

8. It is sometimes said that the main sequence is a mass sequence. What do you think about this? Could you also call it a lifetime sequence?

Discuss your results with your instructor at this point